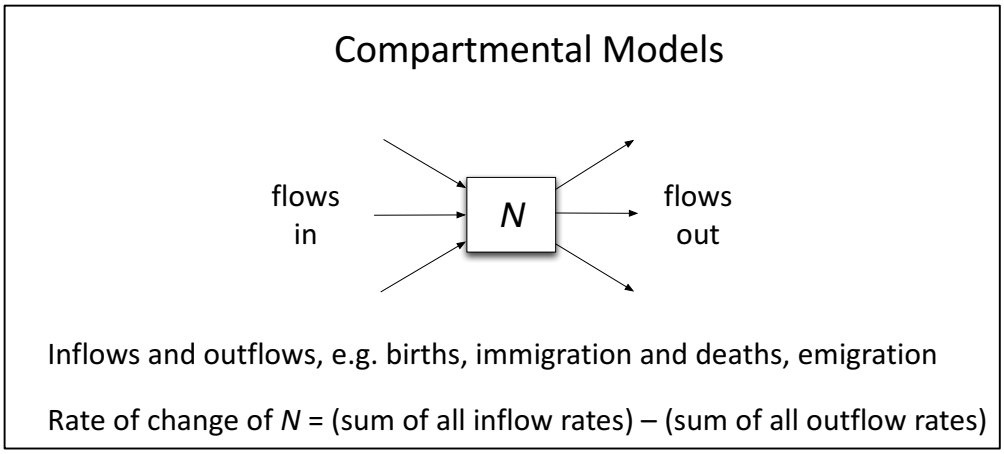
Ingredients that go into models

Example context:



Jargon:

A. The quantities that specify the state of the system are known as **state variables** (e.g. numbers of susceptibles, infectives and recovereds, S, I and R, in the SIR epidemic model).

B. Constants that appear in the rate terms are known as parameters of the model, e.g. k in the first-order kinetics model below

Remember about units: units on the two sides of an equation must be equal. Units are multiplied if two quantities are multiplied, similarly for dividing. Two quantities can only be added or subtracted if they have the same units.

Our model needs to specify the rates of inflows and outflows. The following list talks about some of the flow rates that we see in this course.

0. Zero order kinetics: "Constant rate"

Inflow or outflow occurs at a constant rate.

Example: a doctor administers a drug at a constant rate A. $dX/dt = A$
units of A will equal (units of X)/(units of t)

a population experiences a constant rate a of immigration: $dN/dt = a$

1. First order kinetics: "Rate is proportional to"

Proportional to means "is a multiple of", so "y is proportional to x" means that $y = k\,x$
k is the "constant of proportionality"

Depending on the context, k might be called a rate constant, growth constant, decay constant, or many other things. We typically choose k to be positive.

Examples: Exponential growth, $dN/dt = kN$; Drug metabolism $dX/dt = -kX$
notice the signs of these terms: + denotes inflow, - denotes outflow

units: in both of these examples, k will have units of $(\text{time})^{-1}$

e.g. (units of N)/(units of t) = (units of k) * (units of N)
divide both sides by "units of N" to find units of k

1a. "Rate is proportional to the difference..." or "...proportional to the amount remaining"

"Rate of change of y is proportional to the difference between A and y" means that the rate is a multiple of A-y : $dy/dt = k(A-y)$

In some contexts, e.g. spread of a piece of news, the rate might be proportional to the number of people who have yet to hear the news. If there are N people in the population, and y have heard the news, then $N - y$ have yet to hear the news.
So $dy/dt = k(N-y)$

Units of k here are again 1/(time)

2. "Rate is proportional to both" or "proportional to the product

"Proportional to both y and z" means proportional to the product of y and z.

Classic example here is the spread of an infectious disease from person to person, with the rate of spread being proportional to both the number of susceptible (S) and the number of infectious (I) people

If we write the constant of proportionality as β, the rate of appearance of new infections is βSI

Units of β here are $(\text{time})^{-1}(\text{people})^{-1}$ (using our usual approach to figure units)