BMA 771 Homework Sheet FOUR

To be completed by Friday, September 15th. If you prefer, you can hand your work in at Thursday's class.

Read Sections 10.0 and 10.1 from Strogatz. These talk about difference equations. (In a couple of places, he refers to things we have yet to cover, such as chaos: don't worry about these.)

The Logistic Map:

We will look at the logistic map, which is defined as follows:

$$N_{t+1} = f(N_t) = rN_t(1 - N_t)$$

This is a discrete-time analog of the logistic growth equation.

You should be able to simulate this model, starting from an initial value N_0, using excel or MATLAB. Produce code that can give you both the graph of $f(N)$ vs. N, and a plot of N_t against time. Thinking ahead for the analyses below, you may find it helpful to superimpose a graph of the line $y = N$ on the graph of $f(N)$ vs. N (why?)

We usually restrict attention to $0 \le r \le 4$ and $0 \le N \le 1$. Can you see why? Try setting N_0 outside of this range (pick any r value inside the above interval, such as $r = 1$) and see what happens to the values of N_t. By sketching the graph of $f(N)$, can you explain the restriction on the r values?

Simulate the model for some r values between 0 and 4. Describe some of the behavior you see. Do you see any differences between how the logistic map behaves and the behavior seen in the ODEs in class?

Find the equilibrium points of the model for r values in this range. (One of them may sometimes be negative—even though this is not biologically meaningful, it will be useful to keep track of it.)

Find the stability of these equilibrium points using linear stability analysis. Produce a bifurcation diagram, showing equilibrium points and their stability in terms of r.

Sketch graphs of f that illustrate any changes in behavior—bifurcations—that you find: on a single graph, sketch the curve for r just below, at, and just above the bifurcation point. What property does the graph have at the bifurcation point (i.e. at the equilibrium value of N at the r value when a change happens)?

(The equilibrium analysis relies on the results of a question from the previous homework: in case you had problems there, these results are discussed in section 10.1.)

You should have found two bifurcations. One of these should be familiar from the behavior of ODEs discussed in lectures. The other one should look different to what we have seen before.

How does the system behave for r values just greater than this second bifurcation point? (Do a numerical simulation.)

From Strogatz:

3.1.1, 3.1.3

3.2.1, 3.2.5 (we will likely cover this material on Monday)

Optional (*will not be graded*): The mathematical reduction of systems to "normal form" is discussed in Example 3.2.2 and questions 3.2.6 and 3.2.7.