BMA 771 Homework Sheet THREE

To be completed by Thursday, September 8th. If you prefer, you can hand your work in at Wednesday's class.

Question 1: Linear Difference Equation (again!)

In the lectures, we looked at how to linearize $\frac{dN}{dt} = f(N)$ about an equilibrium point at $N = N^*$, and related the stability of the equilibrium to the derivative of *f* at the equilibrium point.

In this question, we will linearize the analogous difference equation: $N_{t+1} = f(N_t)$.

What condition on *f* gives us an equilibrium point at $N = N^*$? (Be careful: it is not the same condition as we have for an equilibrium of a differential equation.)

As in the lectures, imagine making a small perturbation of *N* about the equilibrium point.

Write $N_t = y_t + N^*$, where y_t is small. Expand *f* about the equilibrium point to give a linear equation for y_{t+1} in terms of y_t.

What condition(s) on the derivative of *f* determine whether the equilibrium point is stable or unstable? Given what we have seen in lectures, what value of the derivative might you expect to see if a bifurcation occurs?

Homework questions from Strogatz:

2.6.1 After answering the question, can you sketch a trajectory in phase space that would depict an oscillation? (Include an arrow on the trajectory to indicate the direction of motion.)

~~3.1.1, 3.1.3~~ (we will likely cover this material on Wednesday)

~~3.2.1, 3.2.5~~

~~3.4.1~~ (we will likely cover pitchfork bifurcations Monday September 11th)~~, 3.4.8~~

Optional (*will not be graded*): If "existence and uniqueness" piques your curiosity, there are some interesting problems from section 2.5, in particular 2.5.1, 2.5.4 and 2.5.6. ~~The mathematical reduction of systems to "normal form" is discussed in Example 3.2.2 and questions 3.2.6 and 3.2.7.~~