BMA 771 Homework Sheet TWO

To be completed by Friday, September 1st. We should have completed all of the material covered on this homework by the time we finish Monday's class.

I always expect you to have read through any handouts and to keep up with where we've got to in Strogatz. By the time this homework is due, we should have completed chapter 2 of the book.

Question 1: Linear Difference Equation

In the lectures, we have focused on ODE models. We showed that the solution of the ODE $\frac{dN}{dt} = rN$, with N equaling $N(0)$ at time $t = 0$ is $N(t) = N(0)e^{rt}$.

The analogous difference equation (i.e. for population values at integer times 0, 1, 2, 3, …) is $N_{t+1} = \lambda N_t$, together with the initial condition that N equals N_0 at time $t = 0$.

What is the analytic solution of this difference equation? (Work out N_1, N_2, N_3, and spot the pattern.) What condition(s) on λ determine whether solutions of the difference equation grow or decay in size over time? (Recall that for the ODE, the behavior depended on whether r was positive or negative.)

Here, we assume λ can be positive or negative, even though a negative value isn't biologically meaningful in a population context (there are other contexts in which it is).

Homework questions from Strogatz:

2.1.1, 2.1.2, 2.1.3

2.2.1 and 2.2.3 (don't worry about trying to solve these ODEs analytically)

2.2.8

2.3.1 (this one will test your calculus skills if they are rusty), and 2.3.4, modified to include an additional condition in the definition of "Allee effect":

In addition to the per-capita growth rate having an intermediate maximum, we require that the per-capita growth rate be negative when the population size is small.

2.4.1, 2.4.4 and 2.4.7.