BMA 771, Fall 2017   Alun L. Lloyd

Homework assignment ONE

Due date: Thursday, August 24th.
(Drop off in my mailbox in 406 Cox Hall by 5pm on Thursday.)

(1) Simple Population Model Sheet (see 'Course Notes' on website)

Read through the sheet and complete the following tasks:

1. Produce **your own code** to simulate the model. (May be easiest in excel, but other packages are acceptable.) Submit (via email) this code as part of your homework.

2. Describe the behavior of the model (using both words and graphs) when there are 40 juveniles and 100 adults at the initial time, the average number of offspring per adult in each time period is 2.5, and the probabilities of different events over the time period are as follows:
   maturation of juvenile 0.4, death of juvenile 0.1, death of adult 0.4.

   From your simulations, can you find the growth rate of the population and the stable age distribution (the relative numbers of juveniles and adults)?

3. Find the (per-capita) growth rate of the population and the stable age distribution by finding the dominant eigenvalue and eigenvector of the matrix that describes the model.

(2) Properties of the Roots of Quadratic Equations

Carry out the questions on the reverse side of this sheet.

Aims of this homework:

1. Learn how to simulate simple difference equation models, producing graphical output.
2. Use simple analysis techniques (e.g. regression routines built in to excel) to interpret this output.
3. Gain some idea of the sorts of behavior exhibited by a simple population model.
4. Check your familiarity with matrix notation and basic linear algebra.
5. The quadratic equations question derives some results that we shall use later on in the course.

(2) Properties of the Roots of a Quadratic Equation

We want to know about the roots (solutions) of the quadratic

$$Ax^2 + Bx + C = 0, \tag{1}$$

where A, B and C are real numbers. In particular, we want to know if the roots are (1) real or complex, and (2) if the roots have negative, positive or zero real part. (When the questions below ask "what can you say about the roots", this is what I am asking about.)

(a) First deal with the case when A is equal to zero. What can you say about the roots of (1) in this case?

From now on, we assume that A is not equal to zero. This means we can divide (1) through by A, giving the following

$$x^2 + bx + c = 0, \tag{2}$$

where $b = B/A$ and $c = C/A$.

In the rest of what follows, we will work with (2) and find the properties of its roots in terms of b and c.

(b) Using the solution to the quadratic equation, find the roots of (2). Under what conditions do we have two real roots, a repeated real root or a pair of complex roots?

(c) Another way to solve the quadratic is to factor. If x_1 and x_2 are the roots, then we must be able to write the quadratic in the form

$$(x - x_1)(x - x_2). \tag{3}$$

Explain why b equals minus the sum of the two roots and c equals the product of the two roots.

(d) What can you say about the roots if $c < 0$? (Are the roots real or complex? What does the information from part c then tell you?)

(e) What can you say about the roots if $c = 0$?

(f) What can you say about the roots if $c > 0$ and $b^2 < 4c$? What if $c > 0$ and $b^2 > 4c$?

(g) Finally, what can you say about the roots in two different special cases: $b = 0$? $b^2 = 4c$?