Potentials (sec 2.7 in Strogatz)

As another way to visualize the dynamics of $dx/dt = f(x)$, Strogatz introduces the potential, $V(x)$, of the system

This quantity is loosely (not exactly!) based on the potential energy of a physical system

Strogatz writes $f = - dV/dx$

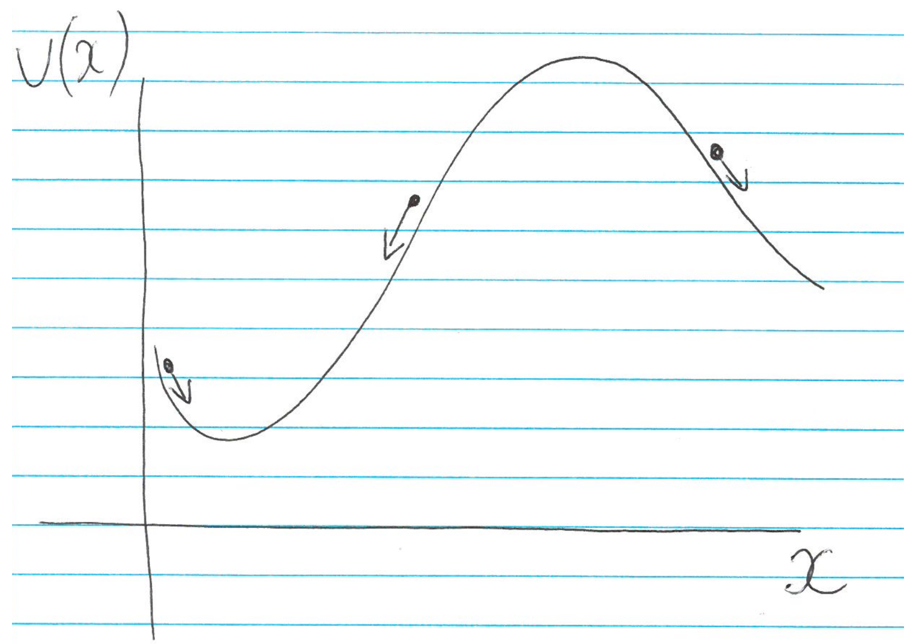
why? In physics, so-called conservative forces [e.g. gravity] can be expressed in terms of a potential function V, with $F = - dV/dx$

Potentials

$dx/dt = - dV/dx$

Velocity of our point x depends on the slope of the potential function

Negative sign means that points move “downhill” along a gradient in the potential function



Potentials

dx/dt = - dV/dx

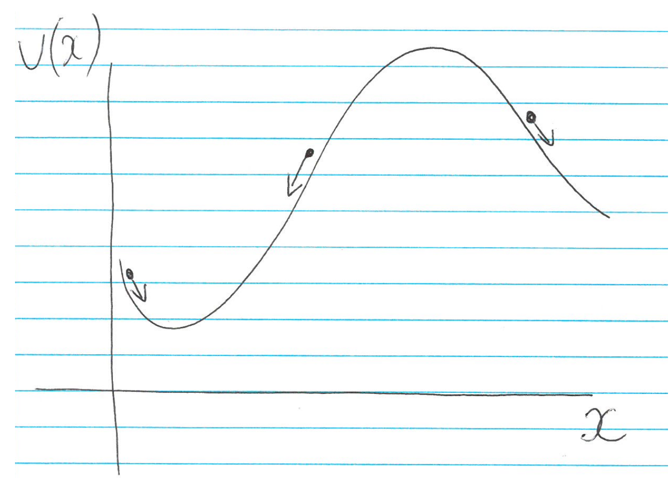
Demonstration that points move downhill: calculate *dV/dt*

$$\frac{dV}{dt} = \frac{dV}{dx}\frac{dx}{dt} \qquad \textit{using chain rule}$$

$$= \frac{dV}{dx} \cdot \left(-\frac{dV}{dx}\right)$$

$$= -\left(\frac{dV}{dx}\right)^2$$

$$\implies \frac{dV}{dt} \leq 0$$



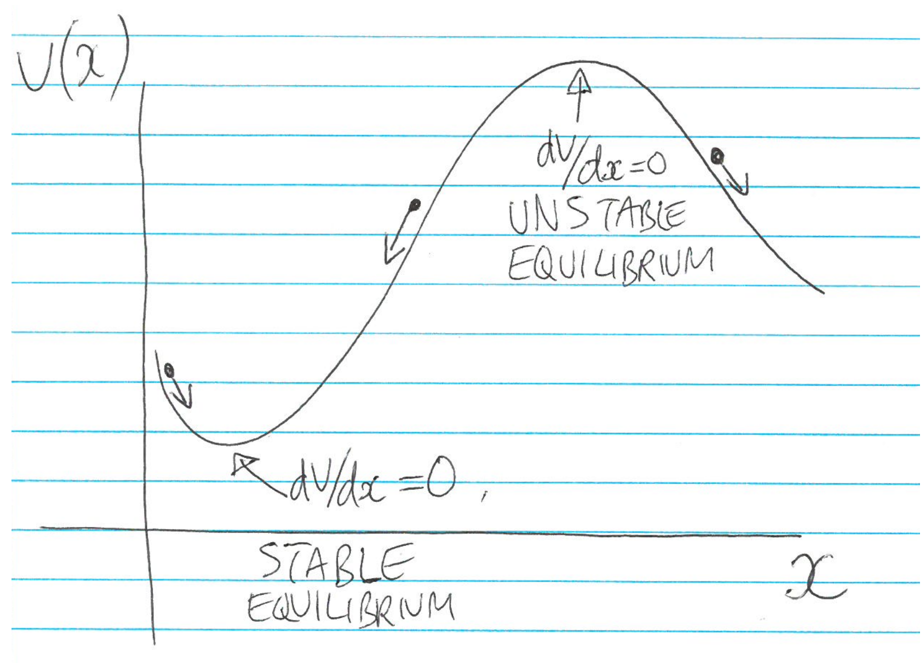
Potentials

$dx/dt = - dV/dx$

Equilibrium whenever $dV/dx = 0$

Stable equilibrium at minimum point of $V(x)$

Unstable equilibrium at maximum point of $V(x)$



About the Analogy…

$dx/dt = f(x)$, $f = - dV/dx$ is not exactly analogous to the physics use of "potential"

In physics, the potential gradient gives the force: $F = - dV/dx$

But Newton's 2nd law says that it is acceleration (not velocity!) that is proportional to force

Strogatz has some complicated, convoluted setup involving highly damped systems that converts a physical law of motion into a first order equation rather than a second order equation (see 2nd part of section 2.6, and references to "goo" in his section 2.7 on potentials)

Potentials

Later in the course, we will use something called **Lyapunov functions**… their setup is somewhat connected to the notion of potential discussed in this video