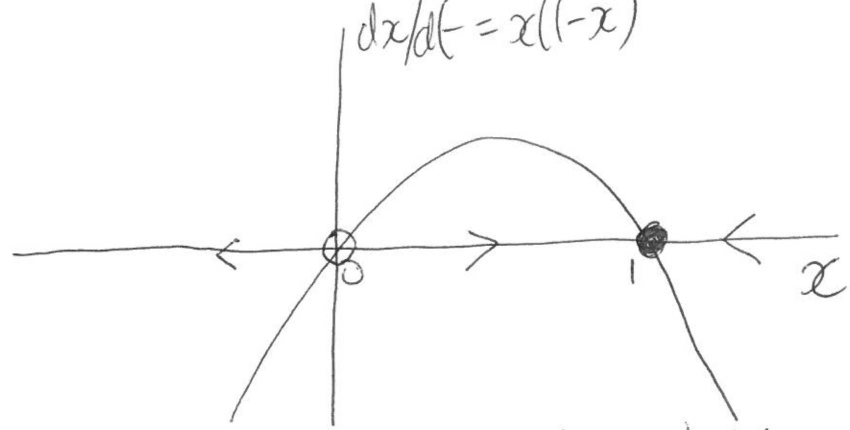
Example that combines geometric, algebraic and potentials approach to finding fixed points and their stability

Find the fixed points of *dx/dt* = *x*(1-*x*) and classify their stability using the three different approaches

① Geometric



$x=0$ unstable eq. , $x=1$ stable eq.

② $f(x) = x(1-x) = x - x^2$ (algebraic)

$f(x) = 0$ when $x = 0$ or $x = 1$

$\frac{df}{dx} = f'(x) = 1 - 2x$

$f'(0) = 1 > 0$ ∴ unstable

$f'(1) = -1 < 0$ ∴ stable

3 Potentials $\quad \frac{dx}{dt} = -\frac{dV}{dx}$

$\frac{dx}{dt} = x(1-x) = x - x^2$

need to find $V(x)$ such that $-\frac{dV}{dx} = x - x^2$

$$\frac{dV}{dx} = -x + x^2$$

$$V(x) = -\frac{1}{2}x^2 + \frac{1}{3}x^3 + C$$

set $C=0$

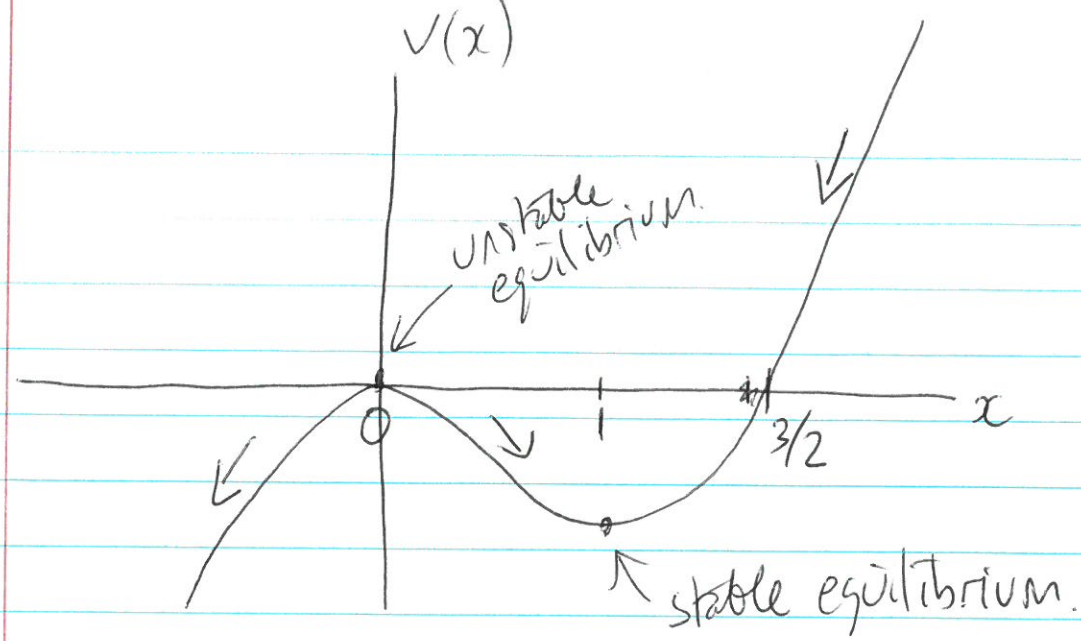
$$V(x) = -\frac{1}{2}x^2 + \frac{1}{3}x^3 = \frac{1}{6}x^2(2x-3)$$

cubic function, coeff of x^3 is positive

$V(x)$ has a double root at $x=0$,
a third root at $x=3/2$

$\frac{dV}{dx} = -x + x^2 = x(x-1)$

local max/min at $x=0$ or $x=1$


V(x)
unstable equilibrium
x
0
1
3/2
stable equilibrium.